Thermodynamic consistency of the equation of state of strongly interacting matter

Jørgen Randrup[1] and Vesa Ruuskanen[2]

[1]*Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720*

[2]*Department of Physics, University of Jyväskylä, Jyväskylä, Finland*

Addressing strongly interacting matter in the region of energy density where the hadronic gas phase coexists with the quark-gluon plasma phase, we have discussed how thermodynamic consistency can be used to constrain the equation of state for uniform matter and we illustrate the method by constructing a T_c-dependent family of thermodynamically consistent equations of state based on simple spline interpolations between the gas and plasma phases [1].

The basic thermodynamic relations between pressure p, temperature T, energy density ε, and entropy density σ are $dp = \sigma dT$ and $d\varepsilon = Td\sigma$. Thus, if the pressure is known as a function of energy density, $p(\varepsilon)$, then $T(\varepsilon)$ can be determined,

$$T(\varepsilon_2) \;=\; T(\varepsilon_1)\exp\left[\int_{\varepsilon_1}^{\varepsilon_2} \frac{dp(\varepsilon)}{d\varepsilon}\frac{d\varepsilon}{\varepsilon+p(\varepsilon)}\right] , \qquad (1)$$

where ε_1 and ε_2 are two arbitrary energy densities. This relation holds generally and it can be utilized to constrain the equation of state whether or not there is a phase transition. From the second Maxwell relation, $\beta d\varepsilon = d\sigma$, we obtain directly a condition on $\beta(\varepsilon) = 1/T(\varepsilon)$,

$$\int_{\varepsilon_1}^{\varepsilon_2} \beta(\varepsilon)d\varepsilon \;=\; \int_{\sigma_1}^{\sigma_2} d\sigma \;=\; \sigma_2-\sigma_1 \,. \qquad (2)$$

When a first-order phase transition is present, the relation $\sigma_Q-\sigma_H = \beta_c(\varepsilon_Q-\varepsilon_H)$ leads to a Maxwell-type condition,

$$\int_{\varepsilon_H}^{\varepsilon_Q} [\beta(\varepsilon)-\beta_c]d\varepsilon \;=\; 0\,, \qquad (3)$$

i.e. the vanishing of the net area between $\beta(\varepsilon)$ and $\beta_c = 1/T_c$.

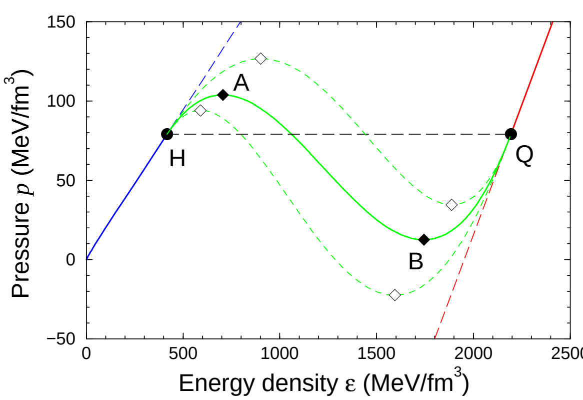

FIG. 1: The equation of state obtained by interpolating between the hadronic gas and the quark-gluon plasma by means of a cubic spline function (solid). Also shown are the equations of state obtained by augmenting the spline with the quartic adjustment using $p_0 = \pm 50\ \mathrm{MeV/fm^3}$ (dashed) (from Ref. [1]).

In order to provide quantitative results, we have, as is common practise, employed ideal gases of either hadrons (and hadron resonances) or quarks and gluons as schematic representations of the two single-phase regimes. We follow the procedure employed in Ref. [2] and consider the cubic spline function $\tilde{p}(\varepsilon)$ between a hadron gas at ε_H and a bag of quarks and gluons at ε_Q, augmented by an adjustable quartic term, $\delta p(\varepsilon) = 16p_0\xi(\varepsilon)^2\bar{\xi}(\varepsilon)^2$, where $\xi \equiv (\varepsilon-\varepsilon_H)/(\varepsilon_Q-\varepsilon_H)$ and $\bar{\xi} = 1-\xi$. By adjusting p_0 it is possible to obtain exact thermodynamic consistency.

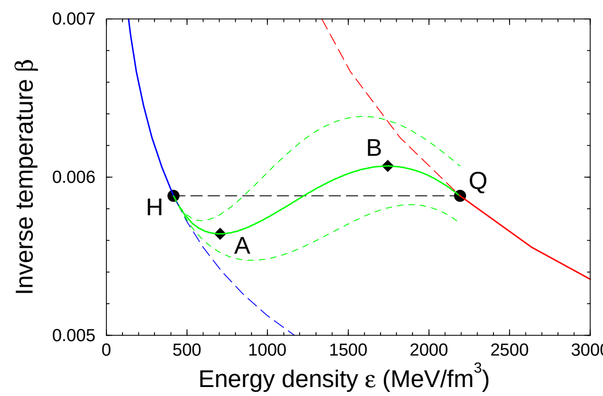

FIG. 2: The inverse temperature $\beta(\varepsilon)$ as obtained by applying the thermodynamic integral relation (1) to the functions $p(\varepsilon)$ shown in Fig. 1, starting from ε_H and ending at ε_Q. (from Ref. [1]). Thus, the cubic equation of state (*i.e.* p_0=0) used in Ref. [2] is to a very good approximation thermodynamically consistent.)

Even though the specific approximation employed may not be accurate in the region of phase change, it may nevertheless be practically very useful for dynamical studies, since the phase-coexistence region has usually been treated only by means of Maxwell's construction as an equilibrium mixed phase at constant pressure, which is a much cruder approach that precludes the study of instability effects. (The procedure should of course be adapted to the presence of chemical potentials.) The present work is thus of particular interest for fluid-dynamical calculations of the rapidly expanding systems formed in high-energy nuclear collisions where the two treatments may yield qualitatively different results. Finally, since the analysis is quite general, the proposed method of approximation may be useful also outside the area of vstrong-interaction physics on which we have focussed here.

[1] J. Randrup and P.V. Ruuskanen, Phys. Rev. C **69**, 047901 (2004).

[2] Jørgen Randrup, Phys. Rev. Lett. **92**, 122301 (2004).